Because the triangle FDE' is isosceles the similar triangle FAC' is isosceles, therefore $AC' = AF$. Through C' draw $C'H'$ parallel with AD and intersecting PP' in H'. Then, by similar triangles, we have,

$$E'F : FD : : E'C' : C'H'.$$

Also $$E'F : FG' : : E'C' : C'B'.$$

But $FG' = FD$ by construction; $\therefore\ C'B' = C'H' = AD$.

[*J. E. Hendricks.*]

EXERCISES.

329

WHAT relations must subsist between the lengths of the edges of a tetrahedron in order that the perpendiculars from the vertices to the opposite sides may meet in a common point? [*Yale Prize Problem.*]

330

FIND the sum of the series

$$1^2 + 3^2 + 6^2 + 10^2 + 15^2 + \ldots + [\tfrac{1}{2}n\,(n+1)]^2.$$

[*Artemas Martin.*]

331

THE extremities of a diameter of a variable ellipse having fixed foci lie on a fixed hyperbola having the same foci; show that the extremities of the conjugate diameter lie on another hyperbola having the same foci.

[*W. Woolsey Johnson.*]

332

FOUR equianharmonic points give four triangles which have four circumcircles. Show that the inverses of any point with regard to these four circles are equianharmonic. [*Frank Morley.*]

333

SHOW that

$$\sin\theta > \theta - \frac{\theta^3}{3!} + \frac{1}{45}\left[\frac{\theta^5}{2^2} - \frac{\theta^7}{2^9} + \ldots (-)^{m+1}\,\frac{\theta^{2m+3}}{2^{\frac{1}{2}(m^2+9m+6)}} \pm \ldots\right];$$

the general term being the mth within the brackets. [*W. H. Echols.*]

334

FIND the necessary relation between the ten distances of five points in space. [*Yale Prize Problem.*]